Fractions

Mathematics Skills Guide

This is one of a series of guides designed to help you increase your confidence in handling mathematics. In this guide you will find help with:

1. What are fractions?
2. Addition and subtraction
3. Multiplication
4. Division

There are often different ways of doing things in mathematics and the methods suggested in the guides may not be the ones you were taught. If you are successful and happy with the methods you use it may not be necessary for you to change them. If you have problems or need help in any part of the work then there are a number of ways you can get help.

For students at the University of Hull

- Ask your lecturers.
- You can contact a math Skills Adviser from the Skills Team on the email shown below.
- Access more maths Skills Guides and resources at the website below.
- Look at one of the many textbooks in the library.
1. What are fractions?

Fractions such as $\frac{1}{2}$, $\frac{3}{7}$, $\frac{9}{25}$ represent part of a unit. Whole numbers can be represented as fractions such as $\frac{2}{1}$, $\frac{16}{4}$, $\frac{90}{10}$ but usually there is no advantage in doing so. The fractions $\frac{1}{2}$, $\frac{3}{7}$, $\frac{9}{25}$ can be represented in diagrams:

- Fraction shaded: $\frac{1}{2}$, $\frac{3}{7}$, $\frac{9}{25}$

The bottom number in a fraction is called the denominator (and shows the number of parts into which the shape is divided) and the top part is called the numerator (and shows the number of parts shaded).

**Proper fractions** are those in which the numerator is less than the denominator (such as $\frac{7}{3}$, $\frac{19}{12}$). Proper fractions represent numbers less than 1.

**Improper fractions** have a numerator larger than the denominator (such as $\frac{23}{19}$) and these represent numbers larger than 1.

**Mixed fractions** are the sum of a whole number with a proper fraction, such as $1 + \frac{4}{19}$, which is written as $1\frac{4}{19}$.

**Equivalent fractions** are fractions with different denominators (and numerators) having the same value. The following diagrams all have the same area shaded although the shapes are not divided up into the same number of pieces. This means that the fractions they represent are equal.

- Equivalent: $\frac{8}{20} = \frac{4}{10} = \frac{2}{5}$

or dividing top and bottom by the same number gives

- Dividing: $\frac{8}{20} = \frac{8 \div 2}{20 \div 2} = \frac{4}{10} = \frac{4 \div 2}{10 \div 2} = \frac{2}{5}$
It is often necessary to convert mixed fractions to improper fractions, and also to convert improper fractions to mixed fractions.

**Worked Examples**

1) Convert $3\frac{2}{5}$ to an improper fraction.

$3\frac{2}{5}$ can be represented as

![Image of fraction representation]

which is equivalent to $\frac{17}{5}$.

In words - each unit is the same as 5 fifths, hence 3 units give 15 fifths. so $3\frac{2}{5}$ gives 15 fifths plus 2 fifths a total of 17 fifths.

This can be written as $3\frac{2}{5} = \frac{3 \times 5 + 2}{5} = \frac{17}{5}$.

2) Convert $6\frac{5}{9}$ to an improper fraction.

As 1 unit is 9 ninths, then 6 units is $6 \times 9 = 54$ ninths and so $6\frac{5}{9}$ is 54 ninths plus 5 ninths, a total of 59 ninths; so $6\frac{5}{9} = \frac{6 \times 9 + 5}{9} = \frac{59}{9}$

3) Convert $\frac{17}{6}$ to a mixed fraction.

$\frac{17}{6}$ can be represented as

![Image of fraction representation]

This is equivalent to $2\frac{5}{6}$

or each group of six $\frac{1}{6}$ ths makes a whole one so 12 lots of $\frac{1}{6}$ th gives 2 whole ones and there are five $\frac{1}{6}$ ths left over, a total of $2\frac{5}{6}$.

or by division $17 \div 6 = 2$ with remainder 5 so $\frac{17}{6} = 2\frac{5}{6}$

**Exercise 1**

1. Convert the following to improper fractions
   a) $1\frac{2}{3}$, b) $3\frac{3}{5}$, c) $5\frac{3}{4}$, d) $2\frac{5}{9}$, e) $11\frac{4}{7}$

2. Convert the following to mixed fractions
   a) $\frac{20}{3}$, b) $\frac{13}{5}$, c) $\frac{33}{4}$, d) $\frac{25}{9}$, e) $\frac{134}{7}$

2. Addition and Subtraction
You may have a calculator that does fractions. Unfortunately in many cases the calculator gives the answer as a decimal rather than as a fraction!! There are at least two ways of adding/subtracting fractions and you are advised to stick to the method you know as long as you are being successful!

No matter which method you use you will have to change the fractions you are adding or subtracting to fractions having the same denominator.

When faced with the question \( \frac{2}{3} + \frac{1}{2} \) some students give the answer as \( \frac{3}{5} \)!

This can be checked by considering a diagram:

\[
\begin{array}{ccc}
\frac{2}{3} & + & \frac{1}{2} \\
\end{array}
\]

In the first 2 columns there are 3 shaded parts but not all of the same size and you can see that \( \frac{2}{3} + \frac{1}{2} \) is greater than 1 so adding the denominators and the numerators together certainly does not give the correct answer. The third column shows \( \frac{3}{5} \) for comparison - you can see that it’s smaller than \( \frac{2}{3} \)!

To do the addition we need to find fractions equivalent to those given but having the same denominators. 3 and 2 both go into a lot of numbers, without leaving a remainder, for instance 120 or 1500 or 30 but the lowest number they both go into is 6 (this is called the Lowest Common Multiple or LCM).

In this case it is 3 \times 2 but if you had 6 and 4 you would use 12 rather than 24.

\[
\frac{2}{3} = \frac{2}{3} \times \frac{2}{2} = \frac{4}{6} \quad \text{and} \quad \frac{1}{2} = \frac{1}{2} \times \frac{3}{3} = \frac{3}{6}
\]

hence \( \frac{2}{3} + \frac{1}{2} = \frac{4}{6} + \frac{3}{6} = \frac{7}{6} = 1 \frac{1}{6} \)

**Examples**
Note – method 1 is the ‘usual’ (and best) method but some people do use method 2!

1. Find the value of \( 1 \frac{4}{9} + 2 \frac{2}{3} \)
Method 1

\[ \frac{4}{9} + 2 \frac{2}{5} = \frac{4}{9} + \frac{2}{5} = \frac{4}{9} + \frac{2}{5} \]

L C M of 5 and 9 is 45, giving

\[ \frac{4}{9} + 2 \frac{2}{5} = \frac{4 \times 5}{9 \times 5} + \frac{2 \times 9}{5 \times 9} = \frac{20 + 18}{45} = \frac{38}{45} \]

It is also a good idea to look at the fractions and estimate what you expect the answer to be so that you can see if your answer is roughly correct. In this example \( \frac{4}{9} + 2 \frac{2}{5} \) must be bigger than 3, it could be close to \( \frac{1}{2} + 2 \frac{1}{2} = 4 \), so the answer of \( \frac{38}{45} \) seems to be about right.

Method 2 (Using improper fractions)

\[ \frac{4}{9} + 2 \frac{2}{5} = \frac{1 \times 9 + 4}{9} + \frac{2 \times 5 + 2}{5} = \frac{13}{9} + \frac{12}{5} \]

L C M of 9 and 5 is 45, giving

\[ \frac{4}{9} + 2 \frac{2}{5} = \frac{13 \times 5}{9 \times 5} + \frac{12 \times 9}{5 \times 9} = \frac{65 + 108}{45} = \frac{173}{45} = \frac{38}{45} \]

2. Find the value of \( 3 \frac{5}{6} - 1 \frac{3}{8} \)

Method 1

\[ 3 \frac{5}{6} - 1 \frac{3}{8} = 3 + \frac{5}{6} - \left( 1 + \frac{3}{8} \right) = 3 + \frac{5}{6} - 1 - \frac{3}{8} = 2 + \frac{5}{6} - \frac{3}{8} \]

L C M of 6 and 8 is 24, so

\[ 3 \frac{5}{6} - 1 \frac{3}{8} = 2 + \frac{5 \times 4}{6 \times 4} - \frac{3 \times 3}{8 \times 3} = 2 + \frac{20}{24} - \frac{9}{24} = 2 \frac{11}{24} \]

It is much safer to do this as one sum and not, as often happens, in two parts. The risk is that that first part is done and the second part gets forgotten!

Method 2

\[ 3 \frac{5}{6} - 1 \frac{3}{8} = \frac{3 \times 6 + 5}{6} - \frac{1 \times 8 + 3}{8} = \frac{23}{6} - \frac{11}{8} \]

L C M of 6 and 8 is 24, so

\[ 3 \frac{5}{6} - 1 \frac{3}{8} = \frac{23 \times 4}{6 \times 4} - \frac{11 \times 3}{8 \times 3} = \frac{92}{24} - \frac{33}{24} = \frac{59}{24} = 2 \frac{11}{24} \]

Estimate: \( 3 \frac{5}{6} - 1 \frac{3}{8} \) must be more than 2 as \( \frac{5}{6} \) is greater than \( \frac{3}{8} \), so the answer of \( 2 \frac{11}{24} \) seems to be of the right size.

3. Find the value of \( 1 \frac{2}{3} - 2 \frac{1}{15} + \frac{4}{5} \)

Method 1

\[ 1 \frac{2}{3} - 2 \frac{1}{15} + \frac{4}{5} = \frac{10}{15} - \frac{2}{15} + \frac{12}{15} = -1 + \frac{10 - 1 + 12}{15} = -1 + \frac{21}{15} = -1 + \frac{6}{15} = \frac{6}{15} = \frac{2}{5} \]

Method 2

\[ 1 \frac{2}{3} - 2 \frac{1}{15} + \frac{4}{5} = \frac{5 \times 31 + 4}{15} = \frac{25 + 31 + 12}{15} = \frac{6}{15} = \frac{2}{5} \]
Estimating the answer to $1\frac{2}{3} - 2\frac{1}{15} + \frac{4}{5}$ is a little more complex. Writing the question as $1\frac{2}{3} + \frac{4}{5} - 2\frac{1}{15}$ we can see that, as $\frac{4}{5}$ is nearly 1, then $1\frac{2}{3} + \frac{4}{5}$ is well over 2 but less than 3 so $1\frac{2}{3} + \frac{4}{5} - 2\frac{1}{15}$ will be positive but less than 1. The answer $\frac{2}{3}$ fits this.

In all cases both methods give the same answer – if they didn’t then one of them must be incorrect.

Exercise 2
Work out the values of the following

1. $\frac{2}{5} + \frac{3}{4}$
2. $\frac{7}{8} - \frac{5}{6}$
3. $\frac{5}{12} + \frac{1}{4}$
4. $1\frac{1}{2} - \frac{3}{4}$
5. $4\frac{1}{6} - 2\frac{2}{3}$
6. $3\frac{4}{5} + 1\frac{1}{4}$
7. $3\frac{1}{2} - 4\frac{1}{4} + 2\frac{3}{4}$
8. $2\frac{4}{9} - 1\frac{2}{3} + \frac{5}{6}$
9. $4\frac{5}{9} - 1\frac{1}{3} - 1\frac{2}{3}$
10. $1\frac{1}{2} - 3\frac{1}{3} + 1\frac{3}{4}$
11. $3\frac{5}{8} - 1\frac{1}{4} - 1\frac{1}{2}$
12. $10\frac{1}{12} - 8\frac{5}{6}$

3. Multiplication

Multiplying and dividing are very different from adding and subtracting and the methods are also very different.

Consider $\frac{2}{3} \times \frac{5}{8}$. This can be read as $\frac{2}{3}$ of $\frac{5}{8}$.

This means having $\frac{5}{8}$ of something, dividing it into 3 parts and taking 2 of them as shown in the diagram.

```
[\ldots]
\frac{5}{8} \text{ shaded}
[\ldots]
\frac{5}{8} \text{ divided into 3}
[\ldots]
\frac{2}{3} \text{ of the } \frac{5}{8} \text{ shaded}
[\ldots]
\text{which is the same as } \frac{10}{24}
```

so $\frac{2}{3} \times \frac{5}{8} = \frac{10}{24}$ which can be seen as $\frac{2}{3} \times \frac{5}{8} = \frac{2 \times 5}{3 \times 8} = \frac{10}{24}$

Multiplying the numerators together gives the numerator of the product and, in the same way, multiplying the denominators together gives the denominator.
Examples

a) Multiply \( \frac{4}{7} \) by \( \frac{5}{8} \):

\[
\frac{4}{7} \times \frac{5}{8} = \frac{4 \times 5}{7 \times 8} = \frac{20}{56} = \frac{5}{14}
\]

This could be checked by using a diagram as above.

b) Multiply \( 1\frac{4}{7} \) by \( \frac{5}{8} \): This can be worked as 1 lot of \( \frac{5}{8} \) plus \( \frac{4}{7} \) of \( \frac{5}{8} \)

which is \( 1 \times \frac{5}{8} + \frac{4}{7} \times \frac{5}{8} = \frac{5}{8} + \frac{20}{56} = \frac{57}{56} \)

This can be done more easily by changing \( 1\frac{4}{7} \) to an improper fraction

\[
\frac{11}{7} \times \frac{5}{8} = \frac{55}{56}
\]

(c) Multiply together \( \frac{4}{7}, \frac{2}{3}, \frac{5}{7} \):

\[
\frac{4}{7} \times \frac{2}{3} \times \frac{5}{7} = \frac{4 \times 2 \times 5}{7 \times 3 \times 7} = \frac{40}{497}
\]

d) Multiply \( 1\frac{4}{7} \) by 4:

\[
1\frac{4}{7} \times 4 = \frac{11}{7} \times 4 = \frac{44}{7} = 6\frac{2}{7}
\]

e) Multiply together \( 2\frac{3}{7}, 1\frac{2}{5}, \frac{1}{4} \):

\[
2\frac{3}{7} \times 1\frac{2}{5} \times \frac{1}{4} = \frac{18}{7} \times \frac{5}{3} \times \frac{1}{4} = \frac{90}{84} = 1\frac{1}{14}
\]

4. Division

12 ÷ 4 can be written as \( \frac{12}{4} \). It means ‘how many lots of 4 are there in 12?’ or ‘how many boxes holding 4kg can I fill from a box holding 12kg?’ - answer 3.

\[
\frac{5}{8} \div \frac{2}{3} \text{ can be written as } \frac{5}{8} \times \frac{3}{2} \text{. It means ‘how many lots of } \frac{2}{3} \text{ are there in } \frac{5}{8} ?' \]

This is not as difficult to work out as it may seem as long as you are careful. We need to do something to make the bottom of the fraction as simple as possible. We know that, as long we multiply or divide the top and bottom numbers of the fraction by the same number we have equivalent fractions of the same value.

If we multiply \( \frac{2}{3} \) by \( \frac{3}{2} \) we get \( \frac{2}{3} \times \frac{3}{2} = \frac{6}{6} = 1 \). So, if we multiply the top and the bottom by \( \frac{3}{2} \) we get

\[
\frac{\frac{5}{8}}{\frac{2}{3}} = \frac{5 \times \frac{3}{2}}{2 \times \frac{3}{2}} = \frac{15}{16} = \frac{15}{16}
\]

In practice we notice that \( \frac{\frac{5}{8}}{\frac{3}{2}} = \frac{5 \times \frac{3}{2}}{1} = \frac{15}{16} \) (multiply the numerator by the inverse of the denominator).

But beware of the trap of “to divide by a fraction turn it upside-down and multiply”. You can’t apply this method directly to problems involving mixed fractions such as \( 3 ÷ 1\frac{2}{6} \).
Note: The fraction $\frac{2}{3}$ is called the (multiplicative) inverse of $\frac{3}{2}$. Any number (except zero) has a multiplicative inverse. Note also that $\frac{2}{3}$ is the multiplicative inverse of $\frac{3}{2}$.

Examples

a) Evaluate $3 \div 1\frac{5}{6}$

To find the (multiplicative) inverse of $1\frac{5}{6}$ we must first express it as an improper fraction:

$$1\frac{5}{6} = \frac{11}{6}$$

so the (multiplicative) inverse of $1\frac{5}{6}$ is $\frac{6}{11}$.

Note that this answer is reasonable as $3 \div 1\frac{5}{6}$ is close to $3 \div 2 = 1\frac{1}{2}$.

b) Evaluate $3\frac{5}{7} \div 4\frac{3}{5}$:

$$3\frac{5}{7} \div 4\frac{3}{5} = \frac{26}{7} \div \frac{23}{5} = \frac{26}{7} \times \frac{5}{23} = \frac{130}{161}$$

c) Evaluate $\frac{7}{12} \div 1\frac{1}{8}$:

there are (at least) two ways of dealing with the next stage

Method 1
multiply, then cancel down
top and bottom have been divided by 4 at the end.

Method 2
cancel down, then multiply
top and bottom have been divided by 4 giving smaller numbers to multiply at the end.

d) Evaluate $15\frac{7}{12} \div 6\frac{7}{8}$

Method 1

$$15\frac{7}{12} \div 6\frac{7}{8} = \frac{187}{12} \div \frac{55}{8} = \frac{187}{12} \times \frac{8}{55} = \frac{187 \times 8}{12 \times 55} = \frac{1496}{660} = \frac{374}{165} = 2\frac{4}{15}$$

Method 2

$$15\frac{7}{12} \div 6\frac{7}{8} = \frac{187}{12} \div \frac{55}{8} = \frac{187}{12} \times \frac{8}{55} = \frac{187 \times 8}{12 \times 55} = \frac{1496}{660} = \frac{374}{165} = 2\frac{4}{15}$$

Note In both cases you divide by 4 and 11.

e) Evaluate $3\frac{1}{3} \times 2\frac{1}{4} \div 7\frac{1}{2} \div \frac{1}{3}$ (Warning – don’t do it in bits!!)

$$3\frac{1}{3} \times 2\frac{1}{4} \div 7\frac{1}{2} \div \frac{1}{3} = \frac{10}{3} \times \frac{9}{4} \div \frac{15}{2} \div \frac{1}{3} = \frac{10}{3} \times \frac{9}{4} \times \frac{2}{15} \times \frac{3}{1} = 3$$

cancelling down by either method should give the same answer!!
Exercise 3
Evaluate the following, writing your answers in as simple a form as possible (cancelled down and as mixed numbers where appropriate).

1. $\frac{7}{8} \times \frac{4}{5}$  
2. $\frac{17}{3} \times \frac{4}{5}$  
3. $6 \div \frac{1}{2}$  
4. $\frac{5}{6} \div \frac{3}{4}$  
5. $2\frac{3}{5} + 3\frac{1}{4}$  
6. $5\frac{1}{7} + 3$
7. $2\frac{1}{7} \times 1\frac{3}{5} \times 2\frac{1}{3}$  
8. $1\frac{5}{9} \times 1\frac{1}{3} \times \frac{3}{4}$  
9. $2\frac{6}{11} \div \frac{7}{11} + 5\frac{2}{5}$  
10. $4\frac{3}{8} \div \left(1\frac{5}{8} - \frac{3}{4}\right)$  
11. $6\frac{2}{3} \times \left(\frac{1}{4} + \frac{1}{3}\right)$  
12. $1\frac{3}{4} + 2\frac{1}{4} \times 2\frac{2}{9} - \left(2\frac{1}{2}\right)^2$

Answers

Exercise 1
1. a) $\frac{5}{9}$, b) $\frac{18}{5}$, c) $\frac{23}{4}$, d) $\frac{23}{9}$, e) $\frac{81}{7}$
2. a) $6\frac{2}{3}$, b) $2\frac{3}{5}$, c) $8\frac{1}{4}$, d) $2\frac{7}{9}$, e) $19\frac{1}{7}$

Exercise 2
1. $1\frac{3}{20}$  
2. $1\frac{1}{24}$  
3. $\frac{2}{3}$  
4. $\frac{3}{4}$  
5. $1\frac{1}{2}$  
6. $5\frac{1}{20}$
7. 2  
8. $1\frac{11}{18}$  
9. $1\frac{5}{9}$  
10. $\frac{1}{20}$  
11. $\frac{7}{8}$  
12. $1\frac{1}{4}$

Exercise 3
1. $\frac{7}{10}$  
2. $1\frac{1}{3}$  
3. 12  
4. $1\frac{1}{9}$  
5. $\frac{4}{5}$  
6. $1\frac{5}{7}$
7. 8  
8. $\frac{7}{8}$  
9. $\frac{20}{27}$  
10. 5  
11. 3  
12. $\frac{1}{2}$

We would appreciate your comments on this worksheet, especially if you’ve found any errors, so that we can improve it for future use. Please contact the Maths Skills Adviser by email at skills@hull.ac.uk
The information in this leaflet can be made available in an alternative format on request using the email above.